

Hall effects on flow of a Williamson fluid through a porous medium in a planar channel with peristalsis

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Abstract— In this paper, the effect of Hall on the flow of Williamson fluid through a porous medium in a planar channel under the assumption of long wavelength is investigated. A Closed form solutions are obtained for axial velocity and pressure gradient. The effects of various emerging parameters on the pressure gradient, time averaged volume flow rate and frictional force are discussed with the aid of graphs.

Keywords: *Hall, Williamson fluid, Hartmann number, long wavelength, peristaltic pumping, Darcy number, porous medium.*

I. INTRODUCTION

The study of peristalsis has received significant consideration in the last few decades primarily because of its significance of Engineering and Biological organisms. More than a few studies have been investigated both theoretical and experimental aspects of the peristaltic flow of Newtonian and non-Newtonian fluids in different situations. In the context of such physiological and industrial applications, the dynamics of peristaltic mechanism has been discussed in detail by various researchers. The non-Newtonian peristaltic flow using a constitutive equation for a second order fluid has been investigated by Siddiqui et al. (1991) for a planar channel and by Siddiqui and Schwarz (1994) for an axisymmetric tube. They have performed a perturbation analysis with a wave number, including curvature and inertia effects and have determined range of validity of their perturbation solutions. The effects of third order fluid on peristaltic transport in a planar channel were studied by Siddiqui et al. (1993) and the corresponding axisymmetric tube results were obtained by Hayat et al. (2002). Haroun (2007) studied peristaltic transport of third order fluid in an asymmetric channel. Subba Reddy et al. (2007) studied the peristaltic flow of a power-law fluid in an asymmetric channel. Peristaltic motion of a Williamson fluid in an asymmetric channel was studied by Nadeem and Akram (2009). Magnetohydrodynamic peristaltic flow of a hyperbolic tangent fluid in a vertical asymmetric channel with heat transfer was studied by Nadeem and Akram (2011). Prasanth Reddy and Subba Reddy (2012) have analyzed the peristaltic pumping of third grade fluid in an asymmetric channel under the effect of magnetic fluid. Effect of hall and ion slip on peristaltic blood flow of Eyring Powell fluid in a non-uniform porous channel was studied by Bhatti et al. (2016). Subba Narasimhudu and Subba Reddy (2017) have studied the Hall effects on the peristaltic flow of a Hyperbolic tangent fluid in a channel. Shalini and Rajasekhar (2019) have investigated the effect of hall on peristaltic flow of a Newtonian fluid through a porous medium in a two-dimensional channel.

Moreover, flow through a porous medium has been studied by a number of researchers employing Darcy's law Scheidegger (1974). Some studies about this point have been given by Varshney (1979) and Raptis and Perdakis (1983). The first study of peristaltic flow

through a porous medium is presented by Elsehawe et al. (1999). Elsehawe et al. (2000) investigated the peristaltic motion of a generalized Newtonian fluid through a porous medium. Hayat et al. (2007) have first investigated the Hall effects on the peristaltic flow of a Maxwell fluid through a porous medium in channel. Peristaltic motion of a Carreau fluid through a porous medium in a channel under the effect of a magnetic field was studied by Sudhakar Reddy et al. (2009). Subba Reddy and Prasanna Reddy (2010) have investigated the effect of variable viscosity on peristaltic flow of a Jeffrey fluid through a porous medium in a planar channel. The peristaltic pumping of Williamson fluid through a porous medium in a horizontal channel with heat transfer was studied by Vasu et al. (2010). Eldabe (2015) have studied the Hall Effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer. Hall effects on the peristaltic transport of Williamson fluid through a porous medium with heat and mass transfer was discussed by Eldabe et al. (2016). Ranjitha and Subba Reddy (2018) have analyzed the radiation effects on the peristaltic flow of a Williamson fluid through a porous medium in a planar channel.

Motivated by these, the effect of Hall on the peristaltic pumping of a Williamson fluid in a planar channel under the assumption of long wavelength is investigated. The expressions for the velocity and axial pressure gradient are obtained by employing perturbation technique. The effects of Weissenberg number, Darcy number, Hall parameter, Hartmann number and amplitude ratio on the axial pressure gradient, time-averaged volume flow rate are analyzed with the help of graphs.

II. MATHEMATICAL FORMULATION

We consider the peristaltic motion of a Williamson fluid through a porous medium in a two-dimensional channel of width $2a$ under the effect of magnetic field. The flow is generated by sinusoidal wave trains propagating with constant speed along the channel walls. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The magnetic Reynolds number is considered small and so induces magnetic field neglected. Fig. 1 represents the physical model of the channel. The wall deformation is given

$$Y = \pm H(X, t) = \pm a \pm b \cos \frac{2\pi}{\lambda}(X - ct) \quad (2.1)$$

where b is the amplitude of the wave, λ the wave length and X and Y - the rectangular coordinates with X measured along the axis of the channel and Y perpendicular to X . Let (U, V) be the velocity components in fixed frame of reference (X, Y) .

The flow is unsteady in the laboratory frame (X, Y) . However, in a co-ordinate system moving with the propagation velocity c (wave frame (x, y)), the boundary shape is stationary. The transformation from fixed frame to wave frame is given by

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad v = V \quad (2.2)$$

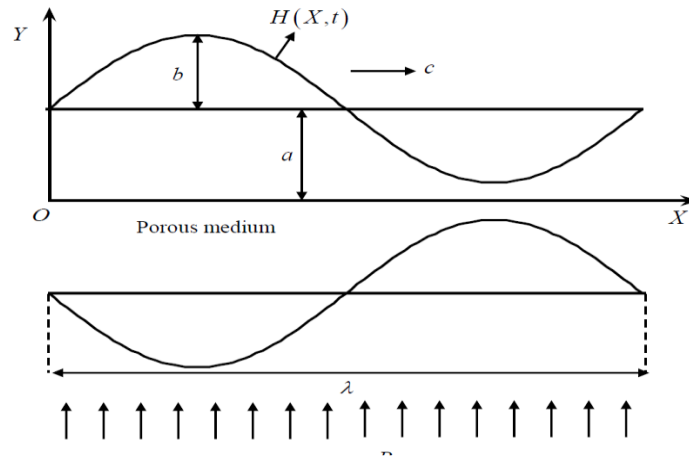


Fig. 1. The physical model

The constitutive equation for a Williamson fluid (given in Vasu et al., 2010) is

$$\tau = - \left[\eta_{\infty} + (\eta_0 + \eta_{\infty})(1 - \Gamma \dot{\gamma})^{-1} \right] \dot{\gamma} \quad (2.3)$$

Where τ is the extra stress tensor, τ_{∞} is the infinite shear rate, τ_0 viscosity is the zero shear rate, Γ is the time constant and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \dot{\gamma}_{ij} \dot{\gamma}_{ji}} = \sqrt{\frac{1}{2} \pi} \quad (2.4)$$

where is the second invariant stress tensor. We consider in the constitutive Equation (2.3) the case for which $\eta_{\infty}=0$ and $\Gamma \dot{\gamma} < 1$ so we can write.

$$\tau = -\eta_0(1 + \Gamma \dot{\gamma}) \dot{\gamma} \quad (2.5)$$

The above model reduces to Newtonian for $\Gamma = 0$

The equations governing the flow in the wave frame of reference are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.6)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} + \frac{\sigma B_0^2}{1+m^2} (mv - (u+c)) - \frac{\eta_0}{k} (u+c) \quad (2.7)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} - \frac{\sigma B_0^2}{1+m^2} (m(u+c) + v) - \frac{\eta_0}{k} v \quad (2.8)$$

where ρ is the density, k is the permeability of the porous medium, σ is the electrical conductivity, B_0 is the magnetic field strength and is the Hall parameter.

The corresponding dimensional boundary conditions are

$$u = -c \text{ at } y = H \text{ (no slip condition)} \quad (2.9)$$

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \text{ (symmetry condition)} \quad (2.10)$$

Introducing the non-dimensional variables defined by

$$\begin{aligned}\bar{x} &= \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{a}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c\delta}, \quad \delta = \frac{a}{\lambda}, \quad \bar{p} = \frac{pa^2}{\eta_0 c \lambda}, \\ \phi &= \frac{b}{a}, \quad h = \frac{H}{a}, \quad \bar{t} = \frac{ct}{\lambda}, \quad \bar{\tau}_{xx} = \frac{\lambda}{\eta_0 c} \tau_{xx}, \quad \bar{\tau}_{xy} = \frac{a}{\eta_0 c} \tau_{xy}, \\ \bar{\tau}_{yy} &= \frac{\lambda}{\eta_0 c} \tau_{yy}, \quad \text{Re} = \frac{\rho ac}{\eta_0}, \quad \text{We} = \frac{\Gamma c}{a}, \quad \dot{\gamma} = \frac{\dot{\gamma} a}{c}, \\ \bar{q} &= \frac{q}{ac}\end{aligned}\tag{2.11}$$

into the Equations (2.6) - (2.8), reduce to (after dropping the bars)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\tag{2.12}$$

$$\text{Re} \delta \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} - \delta^2 \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} +\tag{2.13}$$

$$\begin{aligned}\frac{M^2}{1+m^2} (m\delta v - (u+1)) - \frac{1}{Da} (u+1) \\ \text{Re} \delta^3 \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} - \delta^2 \frac{\partial \tau_{xy}}{\partial y} - \delta \frac{\partial \tau_{yy}}{\partial y} - \\ \frac{\delta M^2}{1+m^2} (m(u+1) + \delta v) - \frac{\delta^2}{Da} v\end{aligned}\tag{2.14}$$

Where $Da = \frac{k}{a^2}$ is the Darcy number

$$\begin{aligned}\tau_{xx} &= -2 \left[1 + \text{We} \dot{\gamma} \right] \frac{\partial u}{\partial x} \\ \tau_{xy} &= - \left[1 + \text{We} \dot{\gamma} \right] \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right) \\ \tau_{yy} &= -2\delta \left[1 + \text{We} \dot{\gamma} \right] \frac{\partial v}{\partial y} \\ \dot{\gamma} &= \left[2\delta^2 \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right)^2 + 2\delta^2 \left(\frac{\partial v}{\partial y} \right)^2 \right]^{\frac{1}{2}} \text{ and}\end{aligned}$$

$M = aB_0 \sqrt{\frac{\sigma}{\eta_0}}$ is the Hartmann number.

Under lubrication approach, neglecting the terms of order δ and Re the Eqs. (2.13) and (2.14) become

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left\{ \left[1 + \text{We} \frac{\partial u}{\partial y} \right] \frac{\partial u}{\partial y} \right\} - \left(\frac{M^2}{1+m^2} + \frac{1}{Da} \right) (u+1)\tag{2.15}$$

$$\frac{\partial p}{\partial y} = 0\tag{2.16}$$

From Eq. (2.15) and (2.16), we get

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + \text{We} \frac{\partial}{\partial y} \left[\left(\frac{\partial u}{\partial y} \right)^2 \right] - \left(\frac{M^2}{1+m^2} + \frac{1}{Da} \right) (u+1)\tag{2.17}$$

The corresponding non-dimensional boundary conditions in the wave frame are given by

$$u = -1 \text{ at } y = h = 1 + \Phi \cos 2\pi x\tag{2.18}$$

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \quad (2.19)$$

The volume flow rate q in a wave frame of reference is given by

$$q = \int_0^h u dy \quad (2.20)$$

The instantaneous flow $Q(X, t)$ in the laboratory frame is

$$Q(X, t) = \int_0^h U dY = \int_0^h (u + 1) dy = q + h \quad (2.21)$$

The time averaged volume flow rate $\bar{Q}$ over one period $T \left(= \frac{\lambda}{c} \right)$ of the peristaltic wave is given by

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1 \quad (2.22)$$

III. Solution

Since Eq. (2.17) is a non-linear differential equation, it is not possible to obtain closed form solution. Therefore we employ regular perturbation to find the solution. For perturbation solution, we expand u , $\frac{dp}{dx}$ and q as follows

$$u = u_0 + We u_1 + o(We^2) \quad (3.1)$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + We \frac{dp_1}{dx} + o(We^2) \quad (3.2)$$

$$q = q_0 + We q_1 + o(We^2) \quad (3.3)$$

Substituting these equations into the Eqs. (2.17) - (2.19), we obtain

3.1 System of order zero We^0

$$\frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} - \left(\frac{M^2}{1+m^2} + \frac{1}{Da} \right) (u_0 + 1) \quad (3.4)$$

and the respective boundary conditions are

$$u_0 = -1 \text{ at } y = h \quad (3.5)$$

$$\frac{\partial u_0}{\partial y} = 0 \text{ at } y = 0 \quad (3.6)$$

3.2 System of order We^1

$$\frac{dp_1}{dx} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial}{\partial y} \left[\left(\frac{\partial u_0}{\partial y} \right)^2 \right] - \left(\frac{M^2}{1+m^2} + \frac{1}{Da} \right) u_1 \quad (3.7)$$

and the respective boundary conditions are

$$u_1 = 0 \text{ at } y = h \quad (3.8)$$

$$\frac{\partial u_1}{\partial y} = 0 \text{ at } y = 0 \quad (3.9)$$

3.3 Solution for system of order We^0

Solving Eq. (3.4) using the boundary conditions (3.5) and (3.6), we obtain

$$u_0 = \frac{1}{N^2} \frac{dp_0}{dx} \left[\frac{\cosh Ny}{\cosh Nh} - 1 \right] - 1 \quad (3.10)$$

Where

$$N = \sqrt{\left(\frac{M^2}{1+m^2} + \frac{1}{Da}\right)}$$

The volume flow rate q_0 is given by $q_0 = \frac{1}{N^3} \frac{dp_0}{dx} \left[\frac{\sinh Nh - Nh \cosh Nh}{\cosh Nh} \right] - h$ (3.11)

From Eq. (3.11), we have

$$\frac{dp_0}{dx} = \frac{N^3 (q_0 + h)}{\tanh Nh - Nh} \quad (3.12)$$

3.4 Solution of order We^1

Substituting Eq. (3.10) in the Eq. (3.7) and solving the Eq. (3.7), using the boundary conditions (3.8) and (3.9), we obtain

$$u_1 = \frac{1}{N^2} \frac{dp_1}{dx} \left[\frac{\cosh Ny}{\cosh Nh} - 1 \right] + \frac{1}{3N^3 \cosh^2 Nh} \left(\frac{dp_0}{dx} \right)^2 (2 \sinh Ny - \sinh 2Ny + A_1 \cosh Ny) \quad (3.13)$$

Where $A_1 = 2 \tanh Nh (\cosh Nh - 1)$

The volume flow rate q_1 is given by

$$q_1 = \frac{1}{N^2} \frac{dp_1}{dx} \left(\frac{1}{N} \tanh Nh - h \right) + \frac{1}{3N^3 \cosh^2 Nh} \left(\frac{dp_0}{dx} \right)^2 A_2 \quad (3.14)$$

Where $A_2 = \frac{2}{N} (\cosh Nh - 1) - \frac{1}{2N} (\cosh 2Nh - 1) + A_1 \frac{\sinh Nh}{N}$

From Eq. (3.14) and (3.12), we have

$$\frac{dp_1}{dx} = \frac{q_1 N^3}{\tanh Nh - Nh} - \frac{A_2 N^6 (q_0 + h)^2}{3 \cosh^2 Nh (\tanh Nh - Nh)^3} \quad (3.15)$$

Substituting Equations (3.12) and (3.15) into the Eq. (3.2) and using the relation

$$\frac{dp_0}{dx} = \frac{dp}{dx} - We \frac{dp_1}{dx} \text{ and neglecting terms greater than } o(We) \text{ we get}$$

$$\frac{dp}{dx} = \frac{(q+h)N^3}{\tanh Nh - Nh} - We \frac{A_2 N^6 (q+h)^2}{3 \cosh^2 Nh (\tanh Nh - Nh)^3} \quad (3.16)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (3.17)$$

Note that, as $M \rightarrow 0$ our results coincide with the results of Vasu et al. (2010) in the absence of heat transfer.

IV Discussions of the Results

Fig. 2 shows the variation of the axial pressure gradient $\frac{dp}{dx}$ with We for $m=0.2$, $M=1$, $Da=0.1$, $\phi=0.5$ and $\bar{Q}=-1$. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing Wiessenberg number We .

The variation of the axial pressure gradient $\frac{dp}{dx}$ with Da for $We=0.01$, $M=1$, $m=0.2$, $\phi=0.5$ and $\bar{Q}=-1$ shown in Fig. 3. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing Darcy number Da .

Fig. 4 depicts the variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $We=0.01$, $M=1$, $Da=0.1$, $\phi=0.5$ and $\bar{Q}=-1$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing Hall parameter m .

The variation of the axial pressure gradient $\frac{dp}{dx}$ with M for $n=0.5$, $m=0.2$, $We=0.01$, $\beta=0.1$, $Da=0.1$, $\phi=0.5$ and $\bar{Q}=-1$ is depicted in Fig. 5. It is observed that, on increasing Hartmann number M increases the axial pressure gradient $\frac{dp}{dx}$.

Fig. 6 shows the variation of the axial pressure gradient $\frac{dp}{dx}$ with ϕ for $m=0.2$, $M=1$, $We=0.01$, $Da=0.1$ and $\bar{Q}=-1$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing amplitude ratio ϕ .

The variation of the pressure rise Δp with $\bar{Q}$ for different values of We with $m=0.2$, $M=1$, $Da=0.1$, and $\phi=0.5$ is shown in Fig. 7. It is noted that, the time-averaged volume flow rate $\bar{Q}$ increases with increasing Wiessenberg number We in pumping ($\Delta p > 0$), free pumping ($\Delta p = 0$) and copumping ($\Delta p < 0$) regions.

Fig. 8 depicts the variation of the Δp with $\bar{Q}$ for different values of Da with $We=0.01$, $m=0.2$, $M=1$ and $\phi=0.5$ is presented in Fig. 11. It is found that, the time-averaged flow rate $\bar{Q}$ decreases with increasing Da in the pumping region, while it increases with increasing Da in both the free pumping and the co-pumping regions.

The variation of the pressure rise Δp with $\bar{Q}$ for different values of m with $We=0.01$, $Da=0.1$, $M=1$ and $\phi=0.5$ is illustrated in Fig. 9. It is observed that, the time-averaged flow rate $\bar{Q}$ decreases with increasing m in the pumping region, while it increases with increasing m in both the free pumping and co-pumping regions.

Fig. 10 depicts the variation of the pressure rise Δp with $\bar{Q}$ for different values of M with $m=0.2$, $Da=0.1$, $We=0.01$, and $\phi=0.5$. It is observed that, the time-averaged flow rate $\bar{Q}$ increases with increasing M in the pumping region, while it decreases with increasing M in both the free-pumping and co-pumping regions.

The variation of the pressure rise Δp with $\bar{Q}$ for different values of ϕ with $m=0.2$, $Da=0.1$, $M=1$, $We=0.01$ is depicted in Fig. 11. It is found that, the time-averaged flow rate $\bar{Q}$ increases with increasing ϕ in both the pumping and free pumping regions, while it decreases with increasing ϕ in the co-pumping region for chosen $\Delta p(<0)$.

V Conclusions

In this chapter, we studied the effects of Hall on the peristaltic flow of a Williamson fluid through a porous medium in a planar channel under the assumption of long wavelength. The expressions for the velocity and axial pressure gradient are obtained by employing perturbation technique. It is found that, the axial pressure gradient and time-averaged flow rate in the pumping region increases with increasing the Weissenberg number We , the Hartmann number M and the amplitude ratio ϕ , while they decreases with increasing Darcy number Da and Hall parameter m .

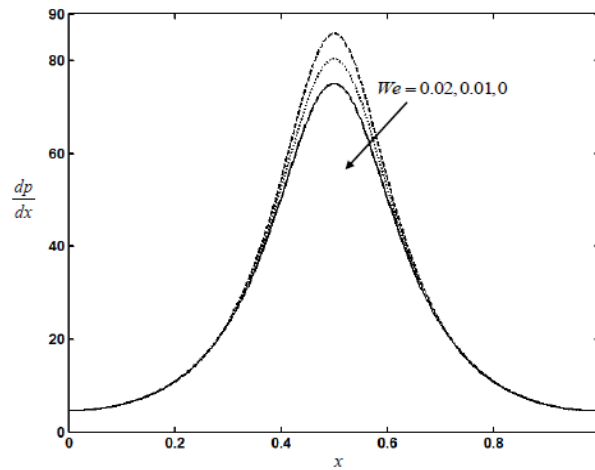


Fig. 2 The variation of the axial pressure gradient $\frac{dp}{dx}$ with We for $m = 0.2$, $M = 1$, $Da = 0.1$, $\phi = 0.5$ and $\bar{Q} = -1$.

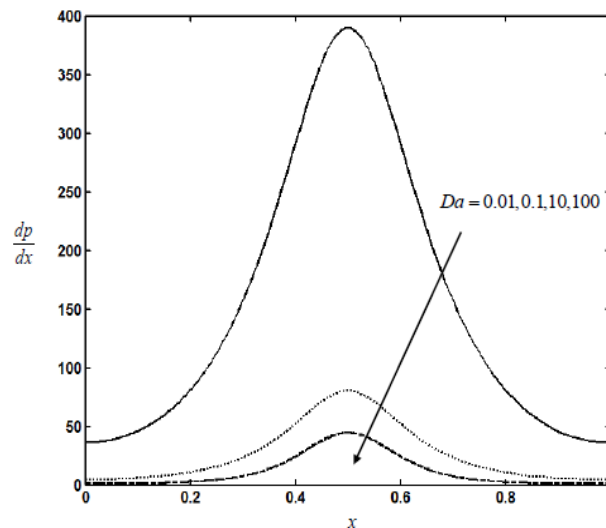


Fig. 3 The variation of the axial pressure gradient $\frac{dp}{dx}$ with Da for $M = 1$, $We = 0.01$, $\phi = 0.5$ and $\bar{Q} = -1$.

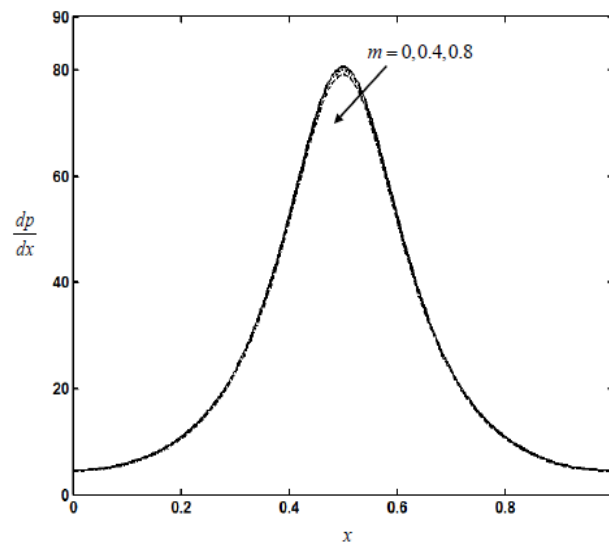


Fig 4 The variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $We = 0.01$, $M = 1$, $Da = 0.1$, $\phi = 0.5$ and $\bar{Q} = -1$.

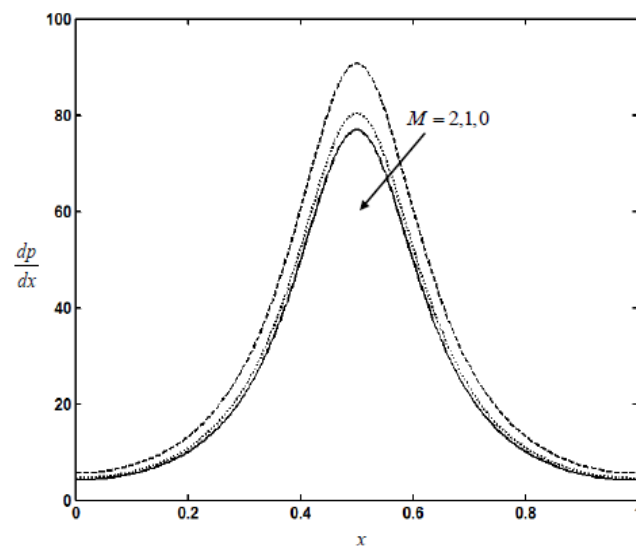


Fig 5 The variation of the axial pressure gradient $\frac{dp}{dx}$ with M for $m = 0.2$, $We = 0.01$, $Da = 0.1$, $\phi = 0.5$ and $\bar{Q} = -1$.

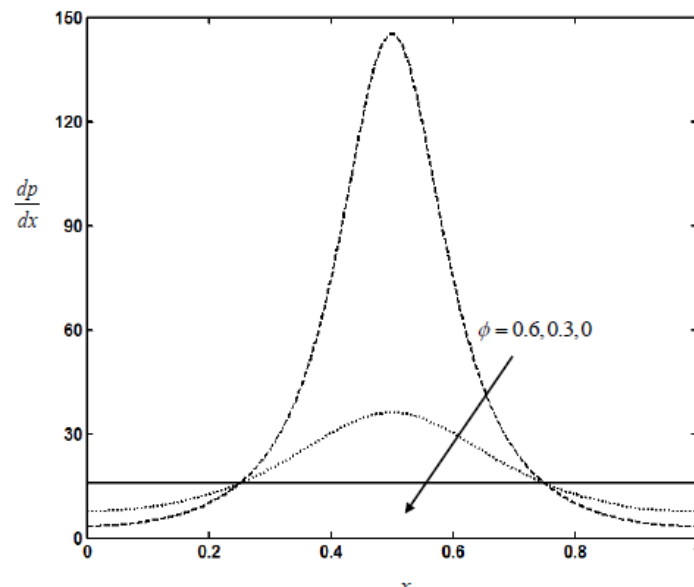


Fig 6 The variation of the axial pressure gradient $\frac{dp}{dx}$ with ϕ for $m=0.2$, $M=1$, $Da=0.1$, $We=0.01$ and $\bar{Q}=-1$.

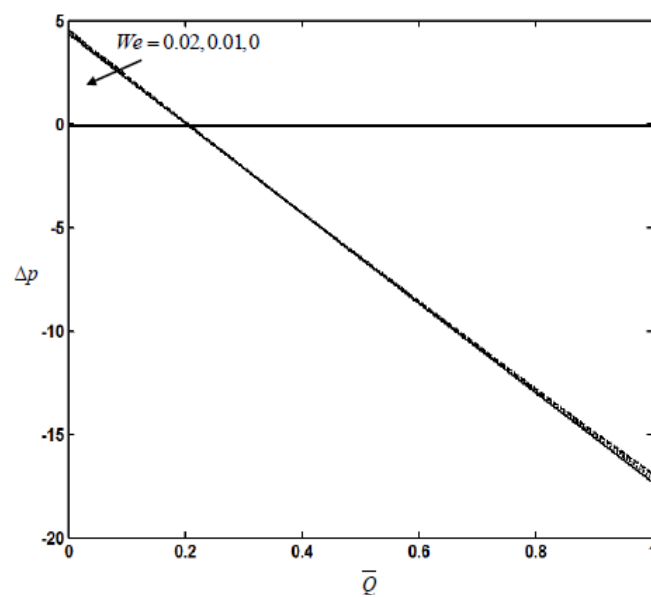


Fig 7 The variation of the pressure rise Δp with $\bar{Q}$ for different values of We with $m=0.2$, $Da=0.1$, $M=1$ and $\phi=0.5$.

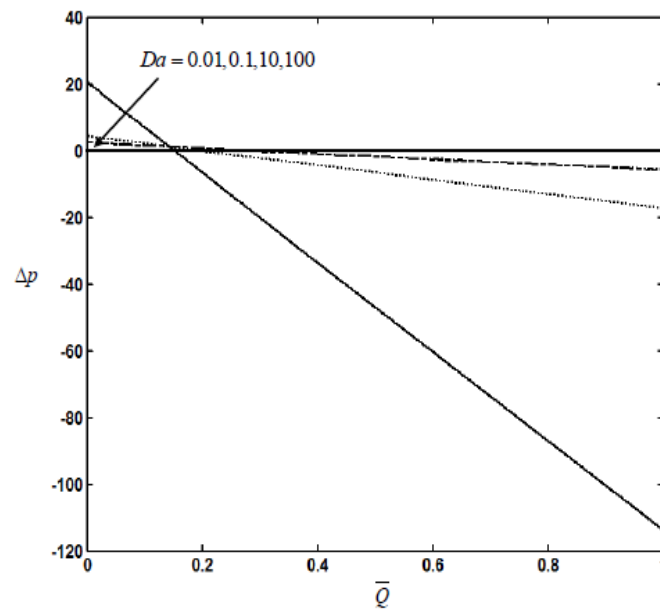


Fig 8 The variation of the pressure rise Δp with $\bar{Q}$ for different values of Da with $m=0.2$, $We=0.01$, $M=1$ and $\phi=0.5$.

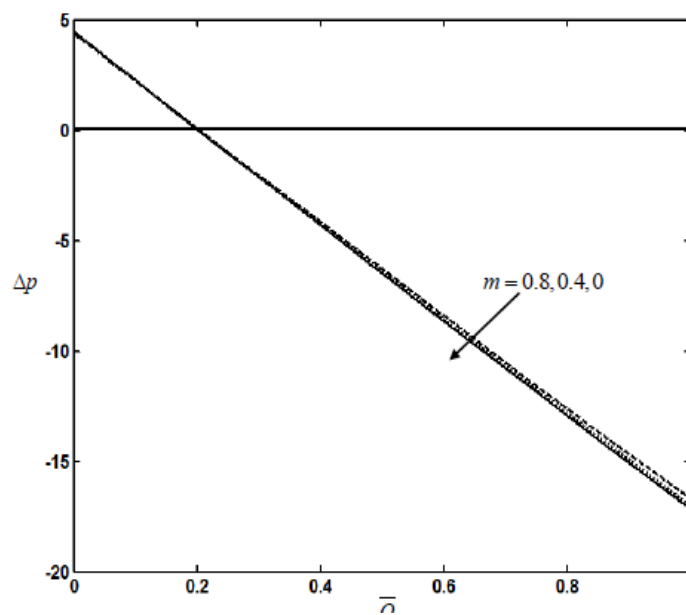


Fig 9 The variation of the pressure rise Δp with $\bar{Q}$ for different values of m with $We=0.01$, $Da=0.1$, $M=1$, and $\phi=0.5$.

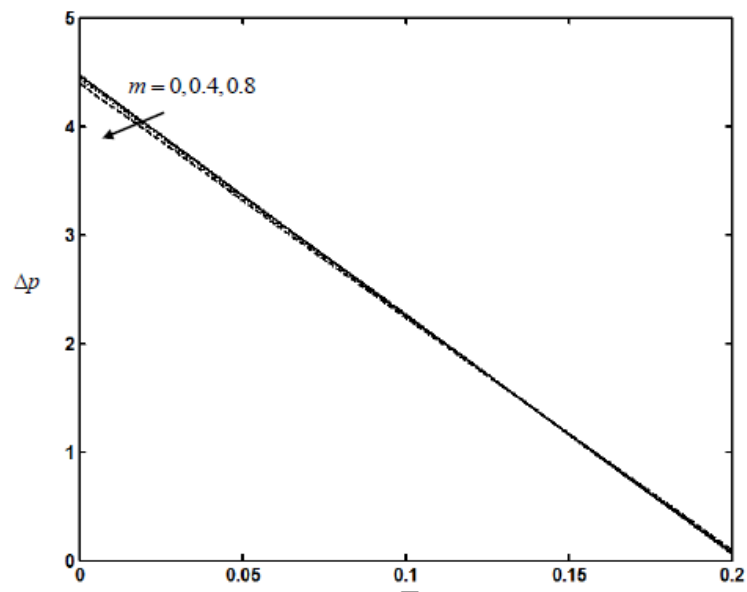


Fig 9(i) Enlargement of fig 9

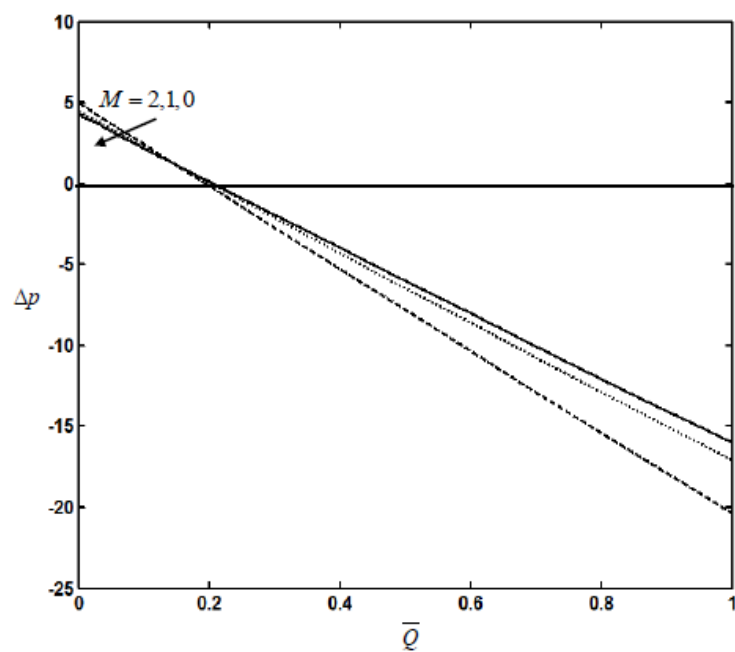


Fig 10 The variation of the pressure rise Δp with $\bar{Q}$ for different values of M with $m = 0.2$, $Da = 0.1$, $We = 0.01$ and $\phi = 0.5$.

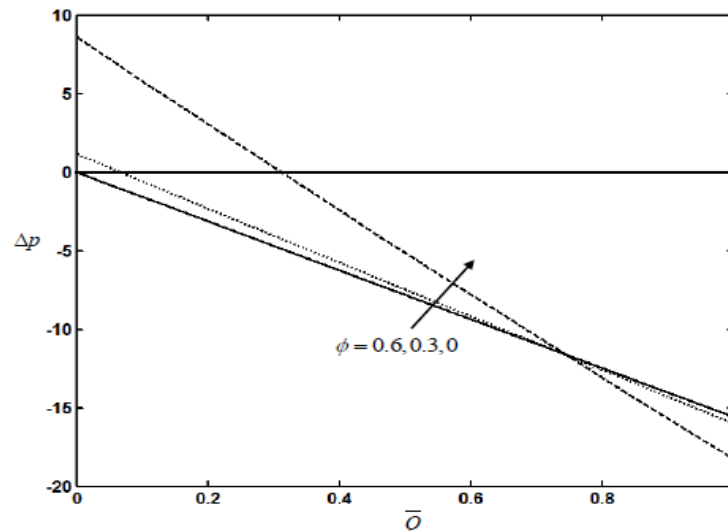


Fig 11 The variation of the pressure rise Δp with $\bar{Q}$ for different values of ϕ with $m=0.2$, $Da=0.1$, $M=1$, and $We=0.01$.

References

- [1] Ai, L. and Vafai, K. An investigation of Stokes' second problem for non-Newtonian fluids, Numerical Heat Transfer, Part A, 47(2005), 955–980.
- [2] Bhatti, M. M., Ali Abbas, M. and Rashidi, M. M. Effect of hall and ion slip on peristaltic blood flow of Eyring Powell fluid in a non-uniform porous channel, World Journal of Modelling and Simulation, Vol. 12 (2016) No. 4, pp. 268-279.
- [3] Eldabe, N.T.M., Ahmed Y. Ghaly, A.Y., Sallam, S.N., Elagamy, K. and Younis, Y.M. Hall effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer, Journal of Applied Mathematics and Physics, 2015, 3, 1138-1150.
- [4] Eldabe, N.T., Elogail, M.A., Elshaboury, S.M. and Hasan, A.A. Hall effects on the peristaltic transport of Williamson fluid through a porous medium with heat and mass transfer, Applied Mathematical Modeling, 40(1)(2016), 315-328.
- [5] Elshahed, M. and Haroun, M. H. Peristaltic transport of Johnson-Segalman fluid under effect of a magnetic field, Math. Probl. Engng, 6 (2005), 663–677
- [6] El Shehawey, E.F., Mekheimer, Kh. S., Kaldas, S. F. and Afifi, N. A. S. Peristaltic transport through a porous medium, J. Biomath., 14 (1999).
- [7] El Shehawey, E.F. and Husseny, S.Z.A. Effects of porous boundaries on peristaltic transport through a porous medium, Acta Mechanica, 143(2000), 165-177.
- [8] Haroun, M.A. Effects of Deborah number and phase difference on peristaltic transport of a third – order fluid in an asymmetric channel. Comm. Non linear sic. Nuber. Simul., 12 (2007), 1464 -1480.
- [9] Hayat, T., Wang, Y., Siddiqui, A.M., Hutter, K. and Asghar, S. Peristaltic transport of a third-order fluid in a circular cylindrical tube, Math. Models & Methods in Appl. Sci., 12(2002), 1691-1706
- [10] Hayat, T and Ali, N. Peristaltically induced motion of a MHD third grade fluid in a deformable tube, Physica A: Statistical Mechanics and its Applications, 370(2006), 225-239.

- [11] Hayat, T., Ali, N, and Asghar, S. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium, *Phys. Letters A*, 363(2007), 397-403.
- [12] Jaffrin, M.Y. and Shapiro, A.H. Peristaltic Pumping, *Ann. Rev. Fluid Mech.*, 3(1971), 13-36.
- [13] Li, A.A., Nesterov, N.I, Malikova, S.N. and Kilatkin, V.A. The use of an impulse magnetic field in the combined of patients with stone fragments in the upper urinary tract. *Vopr kurortol Fizide. Lech Fiz Kult*, 3(1994), 22-24.
- [14] Nadeem, S. and Akram, S. Peristaltic flow of a Williamson fluid in an asymmetric channel, *Comm. Non linear sic. Numer. Simul.*, Vol. 15(7)(2010), 1705-1716.
- [15] Nadeem, S. and Akram, S. Magnetohydrodynamic peristaltic flow of a hyperbolic tangent fluid in a vertical asymmetric channel with heat transfer, *Acta Mech. Sin.*, 27(2) (2011), 237–250
- [16] Nadeem, S. and Akbar, S. Numerical analysis of peristaltic transport of a Tangent hyperbolic fluid in an endoscope, *Journal of Aerospace Engineering*, 24(3) (2011), 309-317.
- [17] Prasanth Reddy, D. and Subba Reddy, M.V. Peristaltic pumping of third grade fluid in an asymmetric channel under the effect of magnetic fluid, *Advances in Applied Science Research*, 3(6)(2012), 3868 – 3877.
- [18] Raptis, A. and Peridikis, C. Flow of a viscous fluid through a porous medium bounded by vertical surface, *Int. J. Engng. Sci.*, 21(1983). 1327-1330.
- [19] Scheidegger, A. E. The physics of through porous media, McGraw-Hill, New York, 1963.
- [20] Shalini, K. and Rajasekhar, K. Peristaltic flow of a Newtonian fluid through a porous medium in a two-dimensional channel with Hall effects, *International Journal of Scientific & Engineering Research Volume 10, Issue 5, May-2019*, 872-877.
- [21] Siddiqui, A.M., Provost, A. and Schwarz, W.H. Peristaltic pumping of a second-order fluid in a planar channel, *Rheol.Acta*, 30(1991), 249-262.
- [22] Siddiqui, A.M., Provost, A. and Schwarz, W.H. Peristaltic pumping of a third-order fluid in a planar channel, *Rheol.Acta*, 32(1993), 47-56
- [23] Siddiqui, A.M. and Schwarz, W.H. Peristaltic flow of a second order fluid in tubes, *J. Non-Newtonian Fluid Mech.*, 53(1994), 257-284.
- [24] Subba Narasimhudu, K. “Effects of hall on peristaltic flows of conducting fluids ”, Ph D., Thesis, Rayalaseema University, (2017).
- [25] Subba Narasimhudu, K. and Subba Reddy, M. V. Hall effects on the peristaltic pumping of a hyperbolic tangent fluid in a planar channel, *Int. J. Mathematical Archive*, 8(3) (2017), 70 - 85.
- [26] Subba Reddy, M.V., Ramachandra Rao, A. and Sreenadh, S. Peristaltic motion of a power law fluid in an asymmetric channel, *Int. J. Non-Linear Mech.*, 42 (2007), 1153 -1161.
- [27] Subba Reddy, M. V., Jayarami Reddy, B. and Prasanth Reddy, D. Peristaltic pumping of Williamson fluid in a horizontal channel under the effect of magnetic field, *International Journal of Fluid Mechanics*, Vol. 3(1)(2011), 89-109.

[28] Subba Reddy, M.V. and Prasanth Reddy, D. Peristaltic pumping of a Jeffrey fluid with variable viscosity through a porous medium in a planar channel, International Journal of Mathematical Archive, 1(2)(2010), 42-54.

[29] Sudhakar Reddy, M., Subba Reddy, M. V. and Ramakrishna, S. Peristaltic motion of a carreau fluid through a porous medium in a channel under the effect of a magnetic field, Far East Journal of Applied Mathematics, 35(2009), 141 – 158.

[30] Varshney, C. L. The fluctuating flow of a viscous fluid through a porous medium bounded by a porous and horizontal surface, Indian. J. Pure and Appl. Math., 10(1979), 1558.

[31] Vasudev, C., Rajeswara Rao, U., Subba Reddy, M.V. and G. Prabhakara Rao, Peristaltic Pumping of Williamson fluid through a porous medium in a horizontal channel with heat transfer, American Journal of Scientific and Industrial Research, 1(3)(2010), 656 - 666.